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RESIDUAL ERRORS.

		Equations.		Direct Calculation.	
		$\cos \delta \,.\, d\alpha$ ″	$d\delta$ ″	$\cos \delta \,.\, d\alpha$ ″	$d\delta$ ″
1857	February 27	−4·8	+2·5	−4·8	+2·5
1858	June 10	−0·1	+0·6	−0·5	+0·6
1859	September 21	+1·1	+0·7	+1·1	+0·7
1861	January 13	+1·1	+1·1	+1·0	+1·3
1862	April 26	+2·4	−0·1	+2·3	−0·1
1864	November 5	+2·7	+0·8	+2·9	+0·8
1866	January 23	+4·2	−1·2	+4·3	−1·1
1867	May 24	−0·8	−0·4	−0·9	−0·3
1868	September 3	−1·2	−0·4	−1·2	−1·1
1869	November 9	−1·0	+0·6	−0·4	+0·7
1871	March 22	+1·3	−0·5	+1·4	−0·6
1872	July 2	−1·0	−0·3	−0·9	−0·1
1873	September 25	−3·7	−1·4	−4·0	−1·5
1875	January 24	−1·5	−0·1	−1·4	0·0
1876	April 30	+0·3	−1·1	+0·3	−1·4

The Centre of Gravity of the Apparent Disk of a Planet.

By Prof. Asaph Hall.

When a planet is observed from the Earth its disk will not be completely illuminated except at the time of opposition. At other times it will appear gibbous or crescent. If we assume the planet to be a sphere, which, according to the best observations is the case with *Mercury*, *Venus*, and *Mars*, the apparent disk will be composed of a semicircle and a semi-ellipse, which are separated by the line of cusps. If a right line be drawn through the centre of the line of cusps and at right angles to it, the centre of gravity of the apparent disk is in this line. In the plane triangle between the Sun, the Earth, and the planet, let ϕ be the angle at the planet, let a be the radius of the semicircular part of the disk, and denote by m the distance of the centre of gravity from the centre of the line of cusps. Then, when the planet is gibbous we shall have

$$m = \frac{8a}{3\pi} \sin^2 \tfrac{1}{2}\, \phi,$$

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and when the planet is crescent,

$$m = \frac{8a}{3\pi} \cos^2 \tfrac{1}{2} \phi,$$

where π is the number 3·1416. The angle ϕ and the radius a can be computed from the data given in the Ephemerides, and the distance m is known, therefore, for any time.

In order to show the use of the quantity m, suppose the angle of position and distance of a satellite of a planet have been observed with a filar micrometer in the following manner. The wire is laid across the apparent disk of the planet in such a way that this disk is divided into two parts of equal area. The wire therefore always passes through the centre of gravity of the apparent disk, and it is necessary, before using the observations for computing orbits, to reduce these measures to the true centre of the planet. If we denote by p and s the observed angle of position and the distance of the satellite, and by θ the angle of position of the line of cusps, the errors of p and s arising from the unsymmetrical form of the disk will be

$$\Delta s = m . \sin (p - \theta),$$

$$\Delta p = \frac{m}{s} . \cos (p - \theta),$$

where Δp is expressed in parts of radius.

To complete the solution we have to find the value of θ. This is given from the spherical triangle between the pole of the Equator and the geocentric places of the Sun and the planet. In this triangle the angle at the planet is $90° - \theta$; and if we denote by d the angular distance between the Sun and the planet, by α, δ, α', δ' the geocentric right ascensions and declinations of the Sun and the planet, we shall have

$$\cos d = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha' - \alpha),$$

$$\sin d \cos \theta = \cos \delta \sin (\alpha' - \alpha),$$

$$\sin d \sin \theta = \sin \delta \cos \delta' - \cos \delta \sin \delta' \cos (\alpha' - \alpha).$$

These equations give the angle θ without ambiguity.

In the case of a spheroidal planet, as *Jupiter* or *Saturn*, the apparent outline of the planet will be composed of ellipses, but the determination of these ellipses, though possible, is much more troublesome, and for these planets the correction for defective illumination is hardly sensible.

Washington, 1877, *November* 15.

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